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APPLICATION LINEAR PROGRAMMING AND OPTIMISATION METHODS TO IMPROVE THE EFFECTIVENESS OF BUDGET EXPEDITURES IN THE FIELD FINANCIAL SUPPORT THE MILITARY TROOPS

B a c k g r o u n d *. The relevance of this article lies in the fact that, in conditions of limited resources, the optimal allocation of financial resources is of great importance for the effective functioning of the state, including military units, as components of the budget system. The purpose of this study is to apply linear programming methods in combination with modern technologies to select the optimal allocation of resources to test a tool that will help institutions, including military units, use financial resources more efficiently and achieve maximum readiness and capabilities with limited budgetary resources. The object of the research. The object of study of this article is the process of financial support of military units as components of the state budget system.*

M e t h o d s *. In the course of the research, the following methods of scientific the following methods of scientific cognition: logical, induction, deduction, classification, grouping - in the study and mechanism of financial support of military units; methods of economic and mathematical modeling, linear programming, formalization - in the formation of the model, its testing on data and development of proposals for its further application.*

R e s u l t s *. The article considers the concept of a military unit from two main aspects: as a military unit and as a budgetary institution managing budgetary funds. Linear programming and modelling methods are used with the help of computer tools and in combination with modern technologies such as Python programming, Google Collaboratory, Jupyter notebook environment, Gurobi optimization library. Based on the results of the study, a financial resource allocation scheme was formed that maximizes the benefit from the use of a limited financial resource.*

C o n c l u s i o n s *. The article explores the possibility of using linear programming methods to determine the optimal allocation of limited financial resources within a military organization among various activities required to meet a specific set of military unit needs, with potential alternative options (groups of options). As a result of the model optimization, the predetermined capabilities of the military unit were satisfied as follows: A – 100%, B – 0%, C – 71%, D – 90%, with the distribution of the financial resource (budget) appearing as follows: A – 29%, B – 0%, C – 37.5%, D – 33.5%, model also ensures 100% utilization of the designated financial resource.*

K e y w o r d s *: budget, defense expenditures, methods of multicriteria optimization, linear programming, efficiency of budget expenditures, economic and mathematical modelling.*

Background

Human endeavors to tackle various problems are almost invariably geared toward discovering the optimal solution. To identify the best choice among all possibilities, it becomes imperative to address problems of maximizing or minimizing certain quantities, that is, seeking the largest or smallest values. Resource constraints often lead to issues of optimization. Optimization problems are frequently encountered in practical domains where state and control variables remain independent of time. Such problems are referred to as static optimization tasks. Solving a static optimization problem involves identifying independent variables that are optimal based on certain predefined criteria.

Linear programming is widely employed in addressing economic challenges, production planning issues, and the management of operational production processes, among others. In many instances, it proves advantageous to transform nonlinear problem formulations into linear ones, thereby leveraging the well-established mathematical framework of linear programming. The essence of linear programming (LP) is framed as follows: it entails determining the maximum or minimum of a linear objective function, with independent variables subject to a set of constraints, including both equalities and inequalities. The number of constraints can be arbitrarily large (Movchan & Stepanets, 2012, p. 115).

Despite certain idiosyncrasies stemming from the distinctive nature of military operations, general economic and financial principles are equally applicable in the domain of financing military troops. To establish the definition of a military unit, which will serve as the foundation for this study, it is worth examining the concept of a military unit from two perspectives. Firstly, a military unit, as an organizational entity, conforms to a standardized structure, operates with dedicated personnel, and is tasked with specific missions. It possesses the necessary resources and capabilities to fulfill its designated functions. Secondly, from a fiscal standpoint, a military unit functions as a budgetary institution in accordance with Ukraine's budget legislation. It is represented by its leader, duly authorized to receive budgetary allocations, enter into budget commitments, and execute budget expenditures.

Consequently, military units are to be regarded henceforth as typical military organizational entities, encompassing both the requisite resources and the means necessary to accomplish their assigned tasks. In the context of this study, we will endeavor to apply linear programming techniques in conjunction with modern technologies to ascertain the optimal allocation of financial resources to meet the operational requirements of a standard military unit.

Literature review. Q. Meng in his study "Benefits distribution and equity in road network" analysed various applications of linear programming. Olayinka and Olusegun studied the prospects of using the linear programming method to maximise profits in the decision-making process of an entrepreneur Oladejo and Abolarinwa confirmed the effectiveness of using linear programming methods in the project optimisation of enterprise profits and at the same time minimising production costs, this study was conducted on the example of a bakery enterprise using AMPL software. The methodology of using linear programming methods and economic and mathematical modelling was formed in textbooks, including those of domestic authors such as A. Movchan, O. Stepanets, K. Mamonov and others. Certain theoretical aspects of the assessment of state budget expenditures are covered in the works of domestic scholars such as L. Lysiak, V. Fedosov, K. Pavliuk, I. Marko, A. Syzov, I. Lyutyi and others.

Methods

The core economic concept of linear programming revolves around the examination of operations associated with determining optimal values for linear variables while operating within resource constraints. Concurrently, mathematical reasoning regarding the interchangeability of factors must always be harmonized with a logically sound comprehension of the nature of the studied phenomenon (Sungatullina, 2014, p. 388).

To address this problem, we will employ methods in computer technology, automation tools, and Python programming elements. The research will be conducted utilizing Google Colaboratory, a cloud-based Jupyter notebook environment. It operates within a web browser and enables individuals with internet access to experiment with machine learning and coding for artificial intelligence. This platform allows users to write and execute Python code, collaborate by sharing and editing code concurrently with other team members, and comprehensively document the entire process by consolidating it into a single notebook containing rich text, charts, images, HTML, and LaTeX (What is Google Colab?, b. d.).

The Gurobi library will serve as a pivotal tool for creating and subsequently optimizing the model. Gurobi (http://www.gurobi.com) is a high-speed mathematical programming solver renowned for its ability to tackle various optimization problems, including linear programming and quadratic programming (The Leader in Decision Intelligence Technology – Gurobi Optimization, b. d.).

Results

IT should be clear that there can be many different tasks, operation concepts of military unit, but to in current research we will try to model predefined example of military unit. It should be noted that by changing the input data, it is possible to further interpolate the model to similar situations, and, if necessary, update the basic constraints and the main function of the model and, thus, adjust its use to the new situation.

Example Optimization Problem. Military unit, A0001, should provide the abilities (A, B, C, D) in the next year. Maintenance of the corresponding abilities requires the expenditure of financial resources, namely: payment of certain goods, works and services to maintain the corresponding characteristics of the carrier ability. The limit of the budget for the next year in the amount of 1 million hryvnias was proved to the military unit.

The leadership of the military unit held a meeting at which it was determined that certain measures must be taken to maintain the capacity carriers at the proper level (carrying out current repairs, modernization, etc.), was created a list of all possible variants measures (from 01 until 28) (purchase of materials, payment for services and works). Weight coefficients were also determined for each measure regarding their impact on the level of readiness of the ability holder. All measures were organized in branches, for each ability only one branch should be taken, shown in Fig. 1.

Ability "A"			Ability "B"			Ability "C"			Ability "D"		
Branch Ba1			Branch Bb1			Branch Bc1			Branch Bd1		
Code	Total price	Weight	Code	Total price	Weight	Code	Total price	Weight	Code	Total price	Weight
1	290 000	0	10	190 000	0	16	250 000	1	18	160 000	0
2	150 000	0	11	250 000	0	17	300 000	1	19	120 000	0
3	170 000	0	12	460 000	0				20	60 000	0
4	60 000	0	13	450 000	0				21	55 000	0
Branch Ba2			Branch Bb2						Branch Bd2		
5	320 000	1	14	420 000	1				22	290 000	1
6	350 000	0	15	310 000	0				23	360 000	0
Branch Ba3									Branch Bd3		
7	100 000	1							Code	Total price	Weight
8	190 000	1							24	260 000	0
Branch Ba4									25	420 000	0
9	350 000	1							26	68 000	0
									27	45 000	0
									Branch Bd4		
									28	220 000	1
									29	215 000	0

Fig. 1. All single measures (expenditures) divided into alternative groups (branches) aimed at ensuring abilities

In order to visualize the set tasks, the following graphic representation of the model of a military unit was created, in the structure of which there are several main abilities. Let's consider the conventional abbreviations that were applied. A conventional abbreviations on the diagram is shown in Fig. 2.

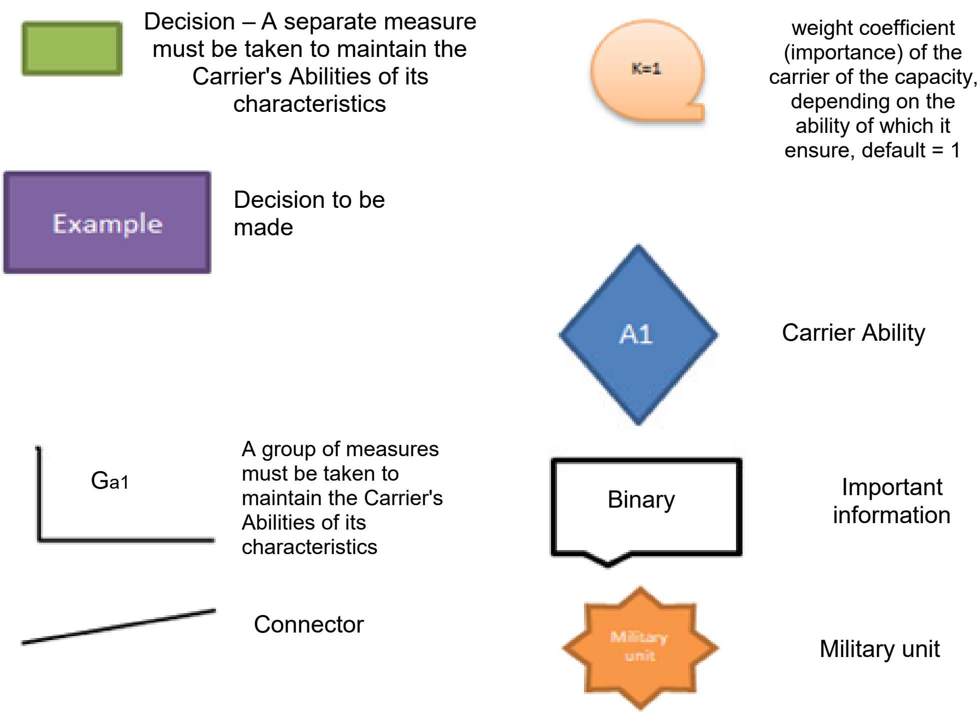

Fig. 2. Conventional graphic symbols used in the visual model of a military unit

The following stages were formed to solve the task

- determination of all Sets and indexes;
- determination of all Parameters;
- determination Decision Variables;
- determination Objective function;
- determination of all Constraints about Objective function;

Sets and indexes

c – index of carrier abilities (from 01 until 28);

n – max index of carrier ability;

$$c \in C = \{1, \dots, n\}; \quad (1)$$

m – index for each expenditure code (single measure); *d* – max index of expenditure code;

$$m \in M = \{1, \dots, d\}; \quad (2)$$

b – index for each branch of measures in carrier ability; *z* – max index branch of measures;

$$b \in B = \{1, \dots, z\}. \quad (3)$$

Parameters

K_c – coefficient weight of the carrier ability with index *c*;

X – Single measure (expenditure code) which have such fields: Price required, Weight coefficient, Price actual, index of branch (*b*) to which this measure belong;

PrR m (Price required) – required budget of expenditure code (single measure) with index *m*;

PrA m (Price actual) – actual budget for Single Measure with index *m*;

Wm – Weight coefficient for single measure with index *m*;

Y – Available year budget.

Decision Variables

PrA – optimal price for single measures (expenditure code) which should be chosen;

b – branch of measures (expenditure code) which should be chosen;

Objective function

$$\max F = \sum_{c=1}^{n} CA_{c*}K_c \quad (4)$$

Constraints

$$CA = \sum_{m=1}^{d} Wm * \left(\frac{PrA\,m}{PrR\,m}\right); \quad (5)$$

PrA (Price actual) – actual budget for Single Measure, according to current financial support, *PrR* (Price required) – required budget of expenditure code (single measure), *W* – Weight coefficient for single measure:

$$PrA \ <= \ PrR. \quad (6)$$

Price actual of single expenditure code must be always less or equal to required price:

$$Y \ge \sum_{m=1}^{d} PrA_{\ m}. \quad (7)$$

The sum of all individual expenditure code (measures) must be less than or equal to the year budget. To solve the problem, computer technology methods, automation tools and Python programming elements will be used. The work will be carried out with the help of Google Colaboratory. Python code and Gurobi library will be used to create the model and its further optimization.

A graphical visualization of the model military unit with 4 main abilities that require financial support is shown in Fig. 3.

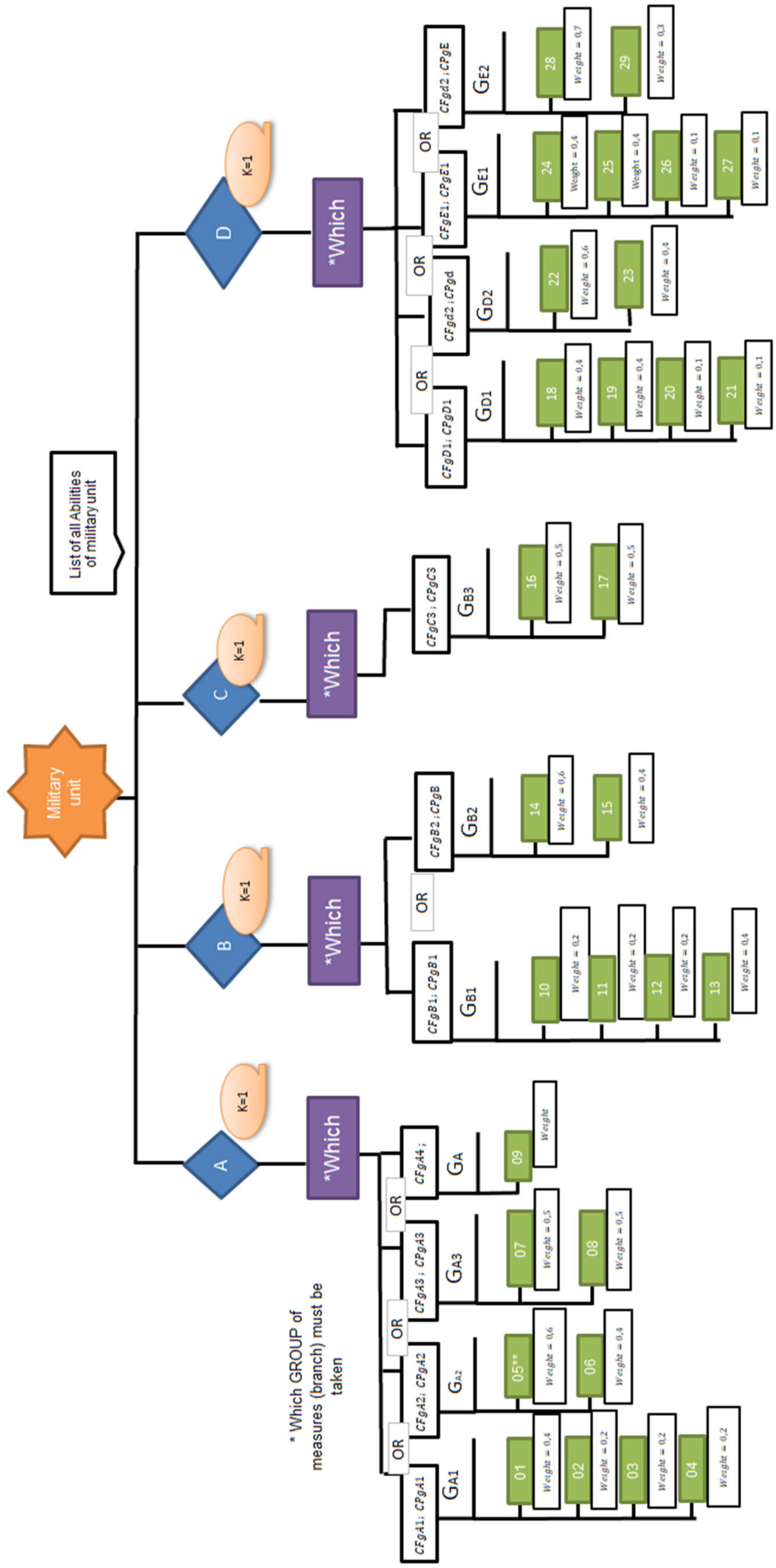

Fig. 3. A graphical visualization of the model of a military unit with 4 main abilities that require financial support

Basic sets and indexes were created in the Google Colaboratory environment, shown in Fig. 4.

```
measures = range(1,30)
priceBudget = pd.Series([290, 150, 170, 60, 320, 350, 100, 190, 350, 190, 250, 460, 450, 420,
                         310, 250, 300, 160, 120, 60, 55, 290, 360, 260, 420, 68, 45, 220, 215],
                         measures, name = "Price_Budget_(PrR)")
veightCoefficient  = pd.Series([0.4, 0.4, 0.2, 0.2, 0.6, 0.4, 0.5, 0.5, 1.0, 0.2, 0.2, 0.2, 0.4,
                                0.6, 0.4, 0.5, 0.5, 0.4, 0.4, 0.1, 0.1, 0.6, 0.4, 0.4, 0.2, 0.3, 0.1, 0.7, 0.3],
                                measures, name = "Weight_coefficient_for_single_measure")
branchIndex = pd.Series(["Ba1", "Ba1", "Ba1", "Ba1", "Ba2", "Ba2", "Ba3", "Ba3", "Ba4", "Bb1", "Bb1",
                          "Bb1", "Bb1", "Bb2", "Bb2", "Bc1", "Bc1", "Bd1", "Bd1", "Bd1", "Bd1", "Bd2",
                          "Bd2", "Bd3", "Bd3", "Bd3", "Bd3", "Bd4", "Bd4" ], measures,
                         name = "Index for branch in CA")
```

Fig. 4. Basic sets and indexes created in the Google Collaboratory environment using the Python programming language

With the help of the Gurobi library, the basic model was created, also the basic function, condition for its maximization and parameters were added to model, Fig. 5.

```
modelo = gp.Model()
x = modelo.addVars(measures, name = "Prica actual", vtype =GRB.INTEGER)

sumaA = modelo.addVars(4, vtype=GRB.BINARY, name="sum alternatives A")
sumaB = modelo.addVars(2, vtype=GRB.BINARY, name="sum alternatives B")
sumaD = modelo.addVars(4, vtype=GRB.BINARY, name="sum alternatives D")

modelo.update()
objetivo = modelo.setObjective(gp.quicksum(veightCoefficient[i]*(x[i]/priceBudget[i]) for i in measures),  GRB.MAXIMIZE)
```

Fig. 5. The main optimization model created in the Google Collaboratory environment using the Python programming language and the Gurobi Optimization library

The formulation of any optimization problem includes conditions which characterize the acceptable values of the variables, and are called constraints tasks. Constraints contain equations of connection between dependent and independent variables in the form of equations as well as functional and parametric constraints in the form of inequalities. Basic restrictions, such as the maximum allowable budget, restrictions on the maximum value of individual expenditures have been added, Fig. 6.

```
max_price_constraint = modelo.addConstrs((x[i]<=priceBudget[i] for i in measures), name="Price actual lover or equal to price required")
max_total_budget_constraint = modelo.addConstr(gp.quicksum(x[i] for i in measures)<=1000, name = "Max year budget")

#for A
modelo.addConstr((sumaA[0]==1)>>((x[5]+x[6]+x[7]+x[8]+x[9])==0))
modelo.addConstr((sumaA[1]==1)>>((x[1]+x[2]+x[3]+x[4]+x[7]+x[8]+x[9])==0))
modelo.addConstr((sumaA[2]==1)>>((x[1]+x[2]+x[3]+x[4]+x[5]+x[6]+x[9])==0))
modelo.addConstr((sumaA[3]==1)>>((x[1]+x[2]+x[3]+x[4]+x[5]+x[6]+x[7]+x[8])==0))

#for B
modelo.addConstr((sumaB[0]==1)>>((x[14]+x[15])==0))
modelo.addConstr((sumaB[1]==1)>>((x[10]+x[11]+x[12]+x[13])==0))

#for D
modelo.addConstr((sumaD[0]==1)>>((x[22]+x[23]+x[24]+x[25])+x[26]+x[27]+x[28]+x[29]==0))
modelo.addConstr((sumaD[1]==1)>>((x[18]+x[19]+x[20]+x[21]+x[24]+x[25])+x[26]+x[27]+x[28]+x[29]==0))
modelo.addConstr((sumaD[2]==1)>>((x[18]+x[19]+x[20]+x[21]+x[22]+x[23]+x[28]+x[29])==0))
modelo.addConstr((sumaD[3]==1)>>((x[18]+x[19]+x[20]+x[21]+x[22]+x[23]+x[24]+x[25])+x[26]+x[27]==0))

modelo.addConstr((sumaA[0] + sumaA[1] + sumaA[2] + sumaA[3]) == 1)
modelo.addConstr((sumaB[0] + sumaB[1]) == 1)
modelo.addConstr((sumaD[0] + sumaD[1] + sumaD[2] + sumaD[3]) == 1)
modelo.update()
modelo.optimize()
```

Fig. 6. The main and other auxiliary model calls created in the Google Collaboratory environment using the Python programming language and the Gurobi Optimization library

Given the possibility of dividing the main (determining) parameter in the model and the number of such options (29), the number of possible existing allocations of financial resources is estimated in thousands. Using the external library of Gurobi Optimization, the calculation was performed in just 2.4 seconds. In the absence of automation elements in this project, a similar search could have taken an incomparably longer period of time.

Based on the results of model optimization, the following distribution of limited financial resources between measures was obtained (Fig. 7), which corresponds to the maximum value of utility. Thus, based on predefined conditions and restrictions, the following distribution between alternative branches of expenditures is optimal.

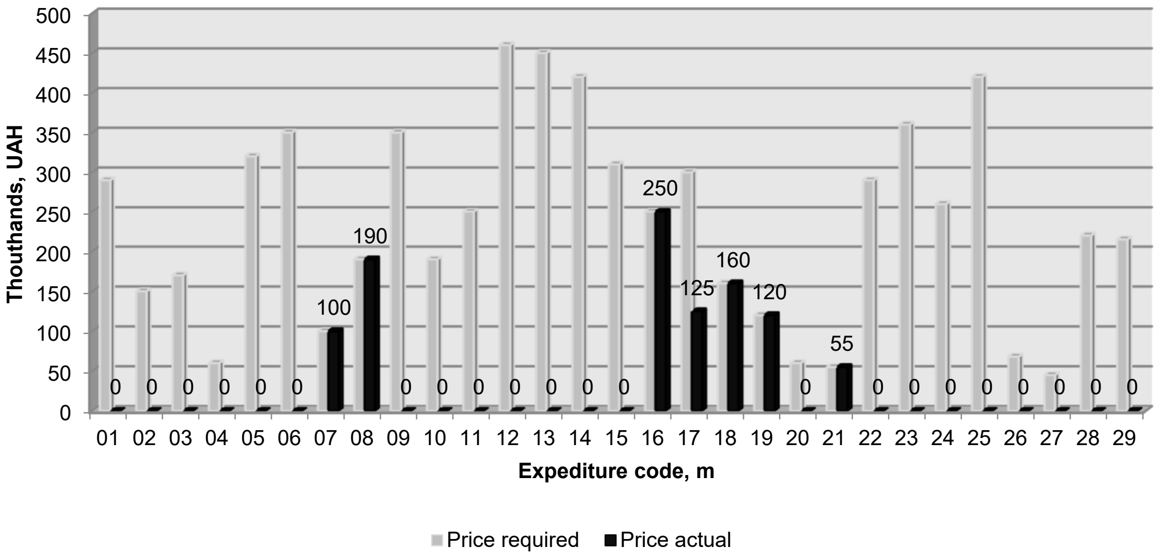

Fig. 7. Distribution of financial resources (budget) between single measures (expenditures), created in accordance with the built optimization model in the Google Collaboratory environment using the Python programming language and the Gurobi Optimization library

Total sum off all selected measures is 1 000 thousands UAH, what is equal to year budget and accept main constraint. Using formula (5), the percentage of meeting the needs of each ability was determined, Fig. 8.

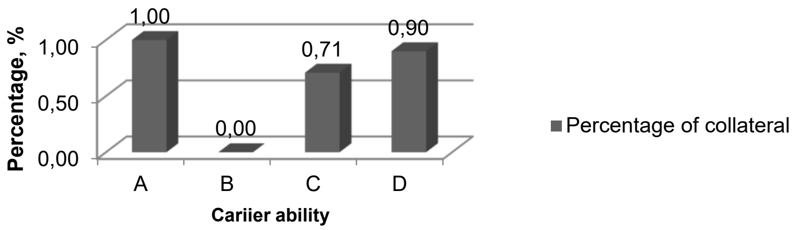

Fig. 8. Status of ensuring that the needs of each ability are met, in %

According to the results of the model optimization, the following distribution of needs between the respective abilities was formed: A is 100 %, B – 0 %, C – 71, D – 90 %. Under the condition of adjusting the limitations of the model, it is possible to change the distribution of financial resources, depending on the specific needs of an individual military unit and the priority of the carriers of abilities, for example, there may be weighting coefficients for all abilities or restrictions on the minimum percentage of ensuring the needs of that or of another bearer of abilities, in the above example, all abilities were assigned a weight coefficient 1.

Discussion and conclusions

Summing up the above, the use of linear programming and optimisation methods in combination with modern computer technologies in the field of financial management has great prospects, including in the field of financial support of troops, because despite certain peculiarities caused by the specifics of military activities, general economic and financial principles are also valid in this area. Given the possibility of dividing the main (determining) parameter in the model and the number of such options (29), the number of possible existing allocations of financial resources is estimated in thousands. The use of linear programming and optimisation methods in combination with modern computer technologies in this way ensures a significant minimisation of the human factor, time savings in choosing the optimal solution, the validity of the choice and the possibility of further expanding the range of data at minimal cost. Further research should focus on expanding the applied model limitations and further automating the process.

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ЗАСТОСУВАННЯ МЕТОДІВ ЛІНІЙНОГО ПРОГРАМУВАННЯ ТА ОПТИМІЗАЦІЇ ДЛЯ ПІДВИЩЕННЯ ЕФЕКТИВНОСТІ БЮДЖЕТНИХ ВИДАТКІВ У СФЕРІ ФІНАНСОВОГО ЗАБЕЗПЕЧЕННЯ ВІЙСЬК

В с т у п . ***Актуальність статті полягає в тому, що в умовах обмеженості ресурсів оптимальний розподіл фінансових ресурсів має велике значення для ефективного функціонування держави, у тому числі військових частин як складових бюджетної системи. Метою дослідження є застосування методів лінійного програмування в поєднанні з сучасними технологіями у виборі оптимального розподілу ресурсів для тестування інструментарію, який допоможе установам більш ефективно використовувати фінансові ресурси і досягати максимальної готовності та забезпечувати спроможності при обмежених бюджетних ресурсах. Об'єктом дослідження є процес фінансового забезпечення військових підрозділів як складових бюджетної системи держави.***

М е т о д и . ***У процесі дослідження для вирішення поставлених завдань використовувались такі методи наукового пізнання: логічний, індукції, дедукції, класифікації, групування – під час вивчення та механізму фінансового забезпечення військових підрозділів; методи економіко-математичного моделювання, лінійного програмування, формалізації – під час формування моделі, її апробації на даних та розроблення пропозицій щодо її подальшого використання.***

Р е з у л ь т а т и . ***Розглянуто поняття військової частини у двох основних аспектах: як військової одиниці та як бюджетної установи, що розпоряджається бюджетними коштами. Використано методи лінійного програмування та моделювання за допомогою комп'ютерних засобів та у поєднанні з такими сучасними технологіями, як програмування мовами Python, Google Collaboratory, середовище Jupyter, бібліотека оптимізації Gurobi. За результатами дослідження сформовано схему розподілу фінансових ресурсів, яка максимізує вигоду від використання обмеженого фінансового ресурсу.***

В и с н о в к и . ***Досліджено можливість використання методів лінійного програмування для визначення оптимального розподілу обмежених фінансових ресурсів військової організації між різними видами діяльності, необхідними для задоволення певного набору потреб військової частини, з можливими альтернативними варіантами (групами варіантів). У результаті оптимізації моделі заздалегідь визначені спроможності військової частини були задоволені таким чином: A – 100%, B – 0%, C – 71%, D – 90%, водночас розподіл фінансового ресурсу (бюджету) мав такий вигляд: A – 29%, B – 0%, C – 37,5%, D – 33,5%. Зауважимо, що модель забезпечує 100% використання визначеного фінансового ресурсу.***

К л ю ч о в і с л о в а : ***бюджет, оборонні видатки, методи багатокритеріальної оптимізації, лінійне програмування, ефективність бюджетних видатків, економіко-математичне моделювання.***

Автор заявляє про відсутність конфлікту інтересів. Спонсори не брали участі в розробленні дослідження; у зборі, аналізі чи інтерпретації даних; у написанні рукопису; в рішенні про публікацію результатів.

The author declares no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses or interpretation of data; in the writing of the manuscript; in the decision to publish the results.